Appendix B Extensions (Online Appendix)

B.1 Selling after no rating

In the base model we focus on the scenario where firm B does not sell after receiving no rating. We establish two results. First, we show that a qualitatively identical equilibrium arises. Second, we show that our comparative statics of $\frac{\partial \delta^*}{\partial \overline{e}}$ are also robust, that is $\frac{\partial \delta^*}{\partial \overline{e}} > 0$.

Note that if firm B does not sell only following a negative rating, this means $E[q_2^B|R=-1] = 0 \leq q^A < E[q_2^B|R=0]$.

Proposition 5. *Suppose the following condition holds:*

$$\frac{q^h - q^A}{q^A} > \frac{(1-\gamma)[\delta^*(1-F(|\overline{p}|)) + (1-\delta^*)(1-F(q^A))]}{\gamma(1-F(q^h-\overline{p}))}. \tag{9}$$

Then in equilibrium both newcomers enter:

1. ***Ratings build reputation:*** $E[q_2^B \mid R=1] > E[q_2^B \mid R=0] > E[q_2^B \mid R=-1]$.
2. ***Ratings are valuable:*** $p_2^B(R=1) = E[q_2^B \mid R=1] - q^A > 0$ *and* $p_2^B(R=0) = E[q_2^B \mid R=0] - q^A > 0$*. If* $R=-1$*, firm* A *sells in period 2.*

Furthermore, in period 1:

3. ***Firm*** A *sets* $p_1^A = 0$ *and faces no demand.*
4. ***Firm*** h *charges* $\overline{p} = \frac{\gamma q^h}{\gamma + (1-\gamma)\delta^*} - q^A$ *with probability 1 and receives* $R \in \{0,1\}$.
5. ***Firm*** l *randomizes over prices in period 1 if* $\frac{\gamma(q^h - 2q^A)}{2(1-\gamma)q^A} > 0$ *and* (11) *hold. In that case:*
   (a) *It charges* $\overline{p} > 0$ *with probability* δ^* *and receives* $R \in \{-1, 0\}$*, where* $\delta^* \in \left(\underline{\delta}, \frac{\gamma(q^h-2q^A)}{2(1-\gamma)q^A}\right)$.
   (b) *It charges* $\underline{p} \equiv -q^A < 0$ *with probability* $1-\delta^*$ *and receives either* $R \in \{0,1\}$.

   Otherwise, firm l *sets* $\delta^* = 1$ *such that* $\overline{p} \leq 0$ *and receives* $R \in \{0,1\}$.

Finally, if (12) *holds, this equilibrium is unique up to off-path beliefs.*

(12) is a technical condition. It ensures that if the mixed-strategy equilibrium is played, it is unique. Otherwise, there might be multiple $\delta^* \in (0,1)$ that induce a mixed-strategy equilibrium.

Recall that our results from Appendix A.1 still apply here, i.e. Lemmas 3 to 9 and Corollary 5 to 8 hold also for the two-period model in which the firm B sells after receiving no rating in period 1. We proceed by establishing Lemmas and Corollaries as we did towards the proof of Proposition 4.

Lemma 14. *Suppose both types of newcomer enter. If* (1)*,* (2) *and* (3) *hold, and* $\overline{p} > 0$*, second* *period beliefs over firm B's quality are*

$$E[q_2^B] = \begin{cases} \frac{\gamma F(q^h - \overline{p}) q^h}{\gamma F(q^h - \overline{p}) + (1-\gamma)(1-\delta^*) F(q^A)} & \text{if } R = 1 \\ \frac{\gamma (1 - F(q^h - \overline{p})) q^h}{\gamma (1 - F(q^h - \overline{p})) + (1-\gamma)[\delta^* (1 - F(\overline{p})) + (1-\delta^*)(1 - F(q^A))]} & \text{if } R = 0 \\ 0 & \text{if } R = -1 \end{cases}$$

where δ^* is the equilibrium probability with which a low-quality firm B plays $\overline{p}$ in period 1.

If (1), (2) *and* (3) *hold, and instead $\overline{p} \leq 0$, second period beliefs are*

$$E[q_2^B] = \begin{cases} \frac{\gamma F(q^h - \overline{p}) q^h}{\gamma F(q^h - \overline{p}) + (1-\gamma) F(|\overline{p}|)} & \text{if } R = 1 \\ \frac{\gamma (1 - F(q^h - \overline{p})) q^h}{\gamma (1 - F(q^h - \overline{p})) + (1-\gamma)(1 - F(|\overline{p}|))} & \text{if } R = 0 \\ 0 & \text{if } R = -1. \end{cases}$$

In either case, ratings are beneficial in equilibrium as $E[q_2^B | R = 1] > E[q_2^B | R = 0] > E[q_2^B | R = -1]$.

If not both types of newcomers enter, consumers believe the quality of newcomers is q^l.

Proof of Lemma 14.
Suppose that both newcomer types enter.

When $\overline{p} > 0$, the argument for the case with $R = 1$ follows directly from Lemma 11. When $\overline{p} \leq 0$, the proofs for $R = 1$ and $R = 0$ follow directly from Lemma 11.

What remains to show is consumer beliefs when $\overline{p} > 0$ for $R = 0$ and $R = -1$, and when $\overline{p} \leq 0$ for $R = -1$.

Suppose first $\overline{p} > 0$. Then when $R = 0$, Corollary 6 and 7 show that both high and low-quality firms can get no rating. This occurs if prices are set sufficiently low but with some probability $1 - F(q^B - \overline{p})$ consumers face a high cost of rating and do not want to rate or in the case of the low-quality firm setting a sufficiently high price with some probability $1 - F(\overline{p})$ consumers face a high cost of ratings and do not want to rate. Therefore, $E[q_2^B | R = 0] = \frac{\gamma (1 - F(q^h - \overline{p})) q^h}{\gamma (1 - F(q^h - \overline{p})) + (1-\gamma)[\delta^* (1 - F(\overline{p})) + (1-\delta^*)(1 - F(q^A))]}$.

Whenever $R = -1$, Corollary 7 shows only low-quality firms may receive bad ratings. In other words, high-quality firms receive bad ratings with probability 0, and $R = -1$ clearly identifies low-quality firms. Therefore, for any p, $E[q_2^B | R = -1] = 0$.

Finally, to show that ratings are beneficial, we have to show $E[q_2^B | R = 1] > E[q_2^B | R = 0] > E[q_2^B | R = -1]$. The second inequality is immediately satisfied. To see the first inequality, consider first $\overline{p} > 0$:

$$\begin{aligned} &\frac{\gamma F(q^h - \overline{p}) q^h}{\gamma F(q^h - \overline{p}) + (1-\gamma)(1-\delta^*) F(q^A)} \\ > &\frac{\gamma(1 - F(q^h - \overline{p})) q^h}{\gamma(1 - F(q^h - \overline{p})) + (1-\gamma)[\delta^*(1 - F(\overline{p})) + (1-\delta^*)(1 - F(q^A))]} \\ &\delta^* F(q^h - \overline{p})(1 - F(\overline{p})) + (1-\delta^*)(F(q^h - \overline{p}) - F(q^A)) > 0. \end{aligned}$$

This inequality always holds because $F(\cdot) \in [0,1]$, $\delta^* \in [0,1]$, $q^h - \overline{p} > q^A$ imply that $F(q^h - \overline{p}) > F(q^A)$. Consider next $\overline{p} \leq 0$:

$$\begin{aligned} \frac{\gamma F(q^h - \overline{p}) q^h}{\gamma F(q^h - \overline{p}) + (1-\gamma) F(|\overline{p}|)} &> \frac{\gamma(1 - F(q^h - \overline{p})) q^h}{\gamma(1 - F(q^h - \overline{p})) + (1-\gamma)(1 - F(|\overline{p}|))} \\ (1 - F(|\overline{p}|)) F(q^h - \overline{p}) q^h) &> (1 - F(q^h - \overline{p})) F(|\overline{p}|) q^h \\ \frac{F(q^h - \overline{p})}{F(|\overline{p}|)} &> \frac{1 - F(q^h - \overline{p})}{1 - F(|\overline{p}|)}. \end{aligned}$$

This inequality always holds as $q^h > 0 \to 1 > F(q^h - \overline{p}) > F(|\overline{p}|) > 0$. Hence, $\frac{F(q^h - \overline{p})}{F(|\overline{p}|)} > 1 > \frac{1 - F(q^h - \overline{p})}{1 - F(|\overline{p}|)}$.

Finally, for the off-equilibrium case after not both types of newcomers enter, consumers believe the quality of newcomers is q^l. By Corollary 5, histories where only the low-quality newcomer enters are off-path. Similarly, if only the high-quality firm enters, beliefs about newcomers are q^h and the newcomer sells at $q^h - q^A$ in each period. But then l has an incentive to deviate and enter and charge $q^h - q^A$ in period 1. □

Lemma 15. *Firm B's decision to sell in period 1 is independent of it's quality realization. Thus, in period 1, a high-quality firm B sells if and only if a low-quality firm B sells. If firm B sells in the second period, it must also sell in the first period.*

Proof of Lemma 15.
First, we know from Corollary 5 that if low-quality firm B sells, a high-quality firm B must also sell.

We now consider firm B selling only when it is high-quality.

Suppose instead firm B only sells when it is high-quality. This means $E[q^B|p] = q^h \; \forall p$. The high-quality firm B receives a positive rating with some positive probability at all prices at which it sells. This allows it to get a continuation profit of $q^h - q^A > 0$ in period 2. But then there is a profitable deviation from a low-quality firm B to enter the market with positive prices in period 1. Hence it cannot be that only the high-quality firm sells in the market in period 1.

Suppose instead a high-quality firm B chooses to enter the market only in period 2 and not period

1. Then it faces the same payoffs as entering in the first period, while forgoing to continuation profits it may get from entering the market in period 1. Thus, h has a profitable deviation to enter in period 1.

Therefore, we can conclude that if firm B enters, it enters the market in the first period and it's entry decision is independent of it's quality realization. $\square$

Lemma 16. *Suppose h and l enter. Both firms receive some positive demand only if firm A sells in period 2, and firm B sells in period 1 with probability* 1. *Firm A sells in period 2 if and only if* $\overline{p} > 0$ *(such that consumers give negative ratings) and firm B sells in period 2 if* (9) *holds.*

Proof of Lemma 16.
The proof takes two parts. First, proving that if firm B enters, it sells in period 1 with probability 1 and firm A only sells in period 2. Second, we derive the conditions for firm A to sell in period 2 and firm B with no rating to sell in period 2.

To show the first statement, recall that by Lemma 15 the low-quality firm B sells in period 1 if and only if the high-quality firm B sells in period 1. Furthermore, by Lemma 15 if firm B sells in period 2, it must also sell in period 1 if it enters. This implies the high-quality firm B must sell in period 1 if it enters. If the high-quality firm B sells in period 1, the low-quality firm B must also sell in period 1. Hence, firm B must sell with probability 1 in period 1 if it enters. This means the only possibility for firm A to sell is in period 2.

Thus, we need to check (i) firm A sells in period 2, (ii) firm B sells in period 2 if it has no rating. To show (i) is immediate as $q^A > 0$ implies that firm A sells if and only if firm B gets a negative rating. Note firm B only gets negative ratings if $\overline{p} > 0$.

To check for (ii) to hold,

$$\frac{\gamma(1-F(q^h-\overline{p}))q^h}{\gamma(1-F(q^h-\overline{p}))+(1-\gamma)[\delta^*(1-F(\overline{p})+(1-\delta^*)(1-F(q^A))]} > q^A$$
$$\Leftrightarrow \frac{q^h-q^A}{q^A} > \frac{(1-\gamma)[\delta^*(1-F(\overline{p}))+(1-\delta^*)(1-F(q^A))]}{\gamma(1-F(q^h-\overline{p}))} \tag{9}$$

$\square$

Corollary 13. *Suppose h and l enter. Ratings are instrumental (i.e. affect beliefs and outcomes) if and only if a high-quality firm B sells in period* 2 *and firm B sells in period 1 with probability* 1. *A high-quality firm B sells in period 2 if* (9) *holds.*

Proof of Corollary 13.
If a high-quality firm B sells in period 2, we know from Lemma 16 that both high- and low-quality firm B must have sold in period 1. Then we also know from Lemma 14 that ratings change consumer beliefs.

In turn, if ratings are instrumental, a high-quality firm B must sell in period 1, which requires that it sells in period 2.

Thus, ratings are instrumental if and only if a high-quality firm B sells in period 2, which holds if $E[q_2^B|R = 1] > q^A$, i.e. if (9) is satisfied. □

Lemma 17. *Suppose* (4) *and* (9) *hold, as well as* (1), (2) *and* (3)*. Then*

1. *If* $\overline{p} > 0$ *and* (11) *hold, there exists a mixed strategies where* $\delta^* \in (0, \frac{\gamma(q^h - 2q^A)}{2q^A(1-\gamma)})$.
2. *If* $\overline{p} > 0$ *and* (11) *hold,* (12) *is a sufficient condition for a unique mixed strategy exists where* $\delta^* \in (0, \frac{\gamma(q^h - 2q^A)}{2q^A(1-\gamma)})$.
3. *If* $\overline{p} > 0$ *holds and* (11) *is violated, then there exists a unique* $\delta^* = 1$.
4. *If* $\overline{p} \leq 0$*, then newcomers sell in period 1 and* $\delta^* = 1$.

Proof of Lemma 17.
We first characterize the total profits that a low-quality firm B would receive if it plays $\overline{p}$ in period 1. Then it's total profits when playing $\underline{p}$ in period 1. We then find the conditions for an interior solution $\delta^* \in (0, 1)$ exists, and when such a solution is unique.

We now characterize the profits the low-quality firm B would receive if it plays $\overline{p}$. When the firm does so, it receives a profit of $\overline{p}$ in the first period, and with probability $F(\overline{p})$ a bad rating and with complementary probability $(1 - F(\overline{p}))$ no rating. This leaves the firm with a continuation profit of 0 with probability $F(\overline{p})$ and $\pi_2(R = 0)$ with probability $(1 - F(\overline{p}))$. Note that the firm is able to sell at a strictly positive price with strictly positive probability and therefore enters.

We next characterize the profits the low-quality firm B would receive if it plays $\underline{p}$. When the firm does so, it receives a profit of $\underline{p}$ in the first period, and with probability $F(-\underline{p}) = F(q^A)$ it receives a good rating and with complementary probability $(1 - F(q^A))$ it receives no rating. Hence leaving the firm with a continuation profit of $\pi_2(R = 1)$ with probability $F(q^A)$ and $\pi_2(R = 0)$ with probability $(1 - F(q^A))$.

Recall from Lemma 7 that $\delta^* = 1$ if $\overline{p} \leq 0$. Additionally, note that by (4), l gets strictly positive demand if it only sells after a positive rating, so this condition implies it also gets strictly positive demand if it sells after a positive and no rating. This proves statement 4. Hence, for the remainder of the proof we focus on the situation where $\overline{p} > 0$.

Note the following 3 conditions must be satisfied for an interior solution $\delta^* \in (0, 1)$ to exist: (i) At any $\delta^* \in (0, 1)$, the low-quality firm B has to be indifferent between setting $\overline{p}$ obtaining either no or bad ratings, and setting $\underline{p}$ obtaining either good or no ratings. (ii) At $\delta^* = 0$, the benefit from setting $\overline{p}$ must be strictly larger than the benefit of setting $\underline{p}$. (iii) At $\delta^* = 1$, the benefit of setting $\underline{p}$ must be strictly larger than the benefit of setting $\overline{p}$.

Thus, for an interior solution, (i) implies the following equation must hold:

$$\begin{aligned}
&\overline{p} + (1 - F(\overline{p}))\pi_2(R = 0) = \underline{p} + F(q^A)\pi_2(R = 1) + (1 - F(q^A))\pi_2(R = 0) \\
&\Leftrightarrow \overline{p} - F(\overline{p})\pi_2(R = 0) = \underline{p} + F(q^A)\pi_2(R = 1) - F(q^A)\pi_2(R = 0) \\
&\Leftrightarrow \frac{\gamma q^h}{\gamma + (1-\gamma)\delta^*} + (F(q^A) - F(\overline{p}))\pi_2(R = 0) - F(q^A)\pi_2(R = 1) = 0, \qquad (10)
\end{aligned}$$

defining the LHS of (10) as K. Where

$$\begin{aligned}
\pi_2(R = 0) &= \frac{\gamma(1 - F(q^h - \overline{p}))q^h}{\gamma(1 - F(q^h - \overline{p})) + (1-\gamma)[\delta^*(1 - F(\overline{p})) + (1 - \delta^*)(1 - F(q^A))]} - q^A, \\
\pi_2(R = 1) &= \frac{\gamma F(q^h - \overline{p})q^h}{\gamma F(q^h - \overline{p}) + (1-\gamma)(1-\delta^*)F(q^A)} - q^A.
\end{aligned}$$

Then (ii) means $K > 0$ at $\delta^* = 0$ is required for an interior solution. Note that at $\delta^* = 0$, $\overline{p} = \frac{\gamma q^h}{\gamma + (1-\gamma)\delta^*} - q^A = q^h - q^A$. Then, evaluated at $\delta^* = 0$

$$\begin{aligned}
K =& q^h - F(q^A)(\frac{\gamma F(q^A)q^h}{\gamma F(q^A) + (1-\gamma)F(q^A)} - q^A) \\
&+ (F(q^A) - F(q^h - q^A))(\frac{\gamma(1 - F(q^A))q^h}{\gamma(1 - F(q^A)) + (1-\gamma)(1 - F(q^A))} - q^A) \\
=& q^h - F(q^A)(\gamma q^h - q^A) + (F(q^A) - F(q^h - q^A))(\gamma q^h - q^A) \\
=& q^h - F(q^h - q^A)(\gamma q^h - q^A),
\end{aligned}$$

because $F(\cdot) \in (0, 1)$ and $q^h > \gamma q^h > 0$, when evaluated at $\delta^* = 0$, $K > 0$.

Then (iii) means $K < 0$ at $\delta^* = 1$ is required for an interior solution. Note that at $\delta^* = 1$, $\overline{p} = \gamma q^h - q^A$. Then, evaluated at $\delta^* = 1$

$$K = \gamma q^h + (F(q^A) - F(\overline{p}))(\frac{\gamma(1 - F(q^h - \overline{p}))q^h}{\gamma(1 - F(q^h - \overline{p})) + (1-\gamma)(1 - F(\overline{p}))} - q^A) < 0, \qquad (11)$$

is required for an interior solution. This implies $F(\overline{p}) > F(q^A)$ is a necessary condition for an interior solution. Note that since $\overline{p}$decreases in δ^*, (11) implies that $F(\overline{p}) > F(q^A)$ for all $\delta^* \in (0, 1]$.

Hence two conditions are required for an interior solution: $\overline{p} > 0$ and (11)$\Rightarrow F(\overline{p}) > F(q^A)$.

To show that the interior solution is unique, it suffices to show that K is decreasing in δ^*.

$$\begin{aligned}
\frac{\partial K}{\partial \delta^*} =& \frac{\partial \overline{p}}{\partial \delta^*} - F(q^A)\frac{\partial \pi_2(R=1)}{\partial \delta^*} - f(\overline{p})\frac{\partial \overline{p}}{\partial \delta^*}\pi_2(R=0) - \frac{\partial \pi_2(R=0)}{\partial \delta^*}(F(\overline{p}) - F(q^A)),\\
\frac{\partial \overline{p}}{\partial \delta^*} =& -\frac{\gamma(1-\gamma)q^h}{(\gamma+(1-\gamma)\delta^*)^2} < 0,\\
\frac{\partial \pi_2(R=1)}{\partial \delta^*} =& \frac{F(q^A)\gamma(1-\gamma)q^h)(F(q^h-\overline{p}) - (1-\delta^*)f(q^h-\overline{p})\frac{\partial \overline{p}}{\partial \delta^*})}{(\gamma F(q^h-\overline{p}) + (1-\gamma)(1-\delta^*)F(q^A))^2} > 0,\\
\frac{\partial \pi_2(R=0)}{\partial \delta^*} =& \frac{\gamma(1-\gamma)q^h\frac{\partial \overline{p}}{\partial \delta^*}(\delta^*(1-F(\overline{p})) + (1-\delta^*)(1-F(q^A)))f(q^h-\overline{p})}{(\gamma(1-F(q^h-\overline{p})) + (1-\gamma)[\delta^*(1-F(\overline{p})) + (1-\delta^*)(1-F(q^A))])^2}\\
&+ \frac{\gamma(1-\gamma)q^h(\delta^* f(\overline{p})\frac{\partial \overline{p}}{\partial \delta^*} + F(\overline{p}) - F(q^A))(1-F(q^h-\overline{p}))}{(\gamma(1-F(q^h-\overline{p})) + (1-\gamma)[\delta^*(1-F(\overline{p})) + (1-\delta^*)(1-F(q^A))])^2}.
\end{aligned}$$

Recall that (11) is required for an interior solution, which implies $F(\overline{p}) > F(q^A)$ for all $\delta^* \in (0,1]$. Then $\frac{\partial \pi_2(R=0)}{\partial \delta^*} > 0$ is a sufficient but not necessary condition for $\frac{\partial K}{\partial \delta^*} < 0$. Rewriting this sufficient condition,

$$\begin{aligned}
\frac{\partial \pi_2(R=0)}{\partial \delta^*} &\geq 0 \Leftrightarrow\\
F(\overline{p}) - F(q^A) &\geq \frac{-\frac{\partial \overline{p}}{\partial \delta^*}((1-F(q^A))f(q^h-\overline{p}) + (1-F(q^h-\overline{p}))\delta^* f(\overline{p}))}{1-F(q^h-\overline{p}) - f(q^h-\overline{p})\frac{\partial \overline{p}}{\partial \delta^*}\delta^*} > 0. \qquad (12)
\end{aligned}$$

Finally, because (11) implies $F(\overline{p}) > F(q^A)$, then it must be that $\overline{p} > q^A$, which means $\frac{\gamma(q^h-2q^A)}{2q^A(1-\gamma)} > \delta^*$.

With these conditions, we may now show the statements in the Lemma.

Statement 1. If $\overline{p} > 0$ and (11) holds, then there exists solutions to (10) such that all solutions are interior, $\delta^* \in (0, \frac{\gamma(q^h-2q^A)}{2q^A(1-\gamma)})$.

Statement 2. A sufficient condition for these interior solutions to be unique is (12).

Statement 3. If $\overline{p} > 0$ and (11) is violated, then there exists a unique $\delta^* = 1$.

Statement 4. If $\overline{p} \leq 0$ there exists a unique $\delta^* = 1$.

□

Proof of Proposition 5.

Suppose (4) and (9) hold, as well as (1), (2) and (3).

Note first that by Lemma 17, (4) and (9) imply that newcomers always have strictly positive demand. Together with our Selection Assumption 1, this implies newcomers enter with probability one.

Statement 1 follows directly from Lemma 14.

Statement 2 follows directly from Corollary 8.

Statement 3 follows directly from Corollary 13.

Statement 4 follows directly from Corollary 6.

The conditions in statement 5 follow from Lemma 17.

The prices in statement 5 follow from the proof of Lemma 17 and the ratings of the low-quality firm from Lemma 7.

The equilibrium level of δ^* and its support follow from Lemma 17.

□

B.1.1 Comparative statics

Again, we focus on mixed-strategy equilibria. We also impose (12) to ensure it is unique. If the mixed-strategy equilibrium is not unique, then we cannot ensure that changes in parameters induce a jump to another mixed-strategy equilibrium.

Corollary 14. $\frac{\partial \delta^*}{\partial \overline{e}} > 0$.

Proof of Corollary 14.
We apply the uniform distribution to (10), which leads to

$$\overline{p} + q^A + \frac{q^A - \overline{p}}{\overline{e}} \pi_2(R=0) - \frac{q^A}{\overline{e}} \pi_2(R=1) = 0.$$

Now, we calculate the total derivative with respect to $\overline{e}$ as

$$-\frac{\gamma(1-\gamma)q^h \frac{\partial \delta^*}{\partial \overline{e}}}{(\gamma + (1-\gamma)\delta^*)^2} + \frac{q^A}{\overline{e}^2}\left[\pi_2(R=1) - \pi_2(R=0)\right] + \frac{q^A}{\overline{e}}\left[\frac{\partial \pi_2(R=0)}{\partial \delta^*} - \frac{\partial \pi_2(R=1)}{\partial \delta^*}\right]\frac{\partial \delta^*}{\partial \overline{e}} - \left[\frac{\partial \overline{p}}{\partial \delta^*}\frac{\partial \delta^*}{\partial \overline{e}} - \frac{\overline{p}}{\overline{e}^2}\right]\pi_2(R=0) - \frac{\partial \pi_2(R=0)}{\partial \delta^*}\frac{\partial \delta^*}{\partial \overline{e}}\frac{\overline{p}}{\overline{e}} = 0.$$

Rearranging leads to

$$\frac{\partial \delta^*}{\partial \overline{e}}\left[\frac{\partial \overline{p}}{\partial \delta^*}\left[1 - \frac{\pi_2(R=0)}{\overline{e}}\right] + \frac{\partial \pi_2(R=0)}{\partial \delta^*}\frac{q^A - \overline{p}}{\overline{e}} - \frac{\partial \pi_2(R=1)}{\partial \delta^*}\frac{q^A}{\overline{e}}\right] = \frac{q^A - \overline{p}}{\overline{e}^2}\pi_2(R=0) - \frac{q^A}{\overline{e}^2}\pi_2(R=1).$$

Recall from the proof of Lemma 17 that $\frac{\partial \overline{p}}{\partial \delta^*} = -\frac{\gamma(1-\gamma)q^h}{(\gamma+(1-\gamma)\delta^*)^2} < 0$, $\frac{\partial \pi_2(R=1)}{\partial \delta^*} > 0$, and by (12) also $\frac{\partial \pi_2(R=0)}{\partial \delta^*} > 0$.

Observe that the right side of this equation is negative. Applying (2), $1 - \frac{\partial \pi_2(R=0)}{\partial \overline{e}} > 0$ and because $\delta^* \in (0, \frac{\gamma(q^h - 2q^A)}{2q^A(1-\gamma)})$, by the proof of Lemma 17, in mixed-strategy equilibria we must have $q^A - \overline{p} < 0$.

Therefore $\frac{\partial \overline{p}}{\partial \delta^*}\left[1 - \frac{\pi_2(R=0)}{\overline{e}}\right] < 0$, $\frac{\partial \pi_2(R=0)}{\partial \delta^*}\frac{q^A - \overline{p}}{\overline{e}} < 0$ and $\frac{\partial \pi_2(R=1)}{\partial \delta^*}\frac{q^A}{\overline{e}} > 0$, which implies the term in the large squared brackets is negative. Hence, $\frac{\partial \delta^*}{\partial \overline{e}} > 0$. □

B.2 Three-period model

We extend our main model and allow for three periods. To start, we explain how we change the main model.

We introduce notation to keep track of histories, denoting $\mathbb{H}_t$ a history of the game until period t, where a history is characterized by the historic ratings until and not including t. To simplify notation, we may suppress the subscript t. We may also use histories as subscripts to clarify arguments, i.e. $\overline{p}_{\{0,1\}}$ is the large newcomer price in period 3 for a history with $R_1 = 0$ and $R_2 = 1$, or $\overline{p}_{\{0\}}$ is the large newcomer price in period 2 for a history with $R_1 = 0$. Throughout, we denote $\emptyset$ as the history in period 1 when the game begins.

Entry. The newcomer B chooses to enter/exit in each periods 1 and 2 and they do so if and only if their subsequent demand is non-zero. Due to this assumption, this extension also captures the possibility of exit better than our baseline model.

Low-quality newcomers enter with probability 1 or 0. We also have to adjust Condition (4) to this setting and replace it with Conditions (17) and (18). The first one ensures that low-quality newcomers enter with probability 1 or 0 in period 1, and the second one ensures this for period 2.

Silence is bad news. We extend Condition (5) to this setting in the following way. We focus on equilibria where firm B sells in period 2 and 3 if and only if it has a history of exclusively good ratings. Again, this is in line with evidence we discuss in the main text that silence is bad news. In other words, firm B only sells if $\mathbb{H}_t \in \{\emptyset, \{1\}, \{1,1\}\}$. A sufficient condition is that $\max\{E[q_3^B|\{1,0\}], E[q_2^B|\{0\}]\} < q^A$. I.e. if the newcomer does not sell after a history of one good and no rating, and not after a history of no rating, they also do not sell after other histories that involve no rating or a negative rating, since expectations must be lower in such histories. Intuitively, this is satisfied if q^h is sufficiently large, since then the high-quality firm is unlikely to receive no rating, lowering expectations for every history with no rating.

Other conditions and restrictions. The conditions that the PDF of F is sufficiently flat are the same as in the proof of Proposition 4. Also the equilibrium-selection assumptions are the same as in the main text. We also continue to assume $\gamma q^h < q^A$.

Proposition 6. *If f is sufficiently flat (i.e.* (1)*,* (2) *and* (3) *hold),* $\max\{E[q_3^B|\{1,0\}], E[q_2^B|\{0\}]\} < q^A$*, and* (17) *and* (18) *hold. There exists an equilibrium that is unique up to off-path beliefs. In this equilibrium, there exist unique values* $\underline{\delta}_\emptyset \in (0,1)$ *and* $\underline{\delta}_{\{1\}} \in (0,1)$ *such that the following holds.*

Both types of newcomer B enter in period 1 if and only if $\delta^*_\emptyset \geq \underline{\delta}_\emptyset$; otherwise none enters. Both types of newcomer B enter in period 2 if and only if $\delta^*_\emptyset \geq \underline{\delta}_\emptyset$ and $\delta^*_{\{1\}} \geq \underline{\delta}_{\{1\}}$ hold; otherwise none enters in period 2.

Furthermore, if the newcomer enters in period t:

1. ***Ratings build reputation:*** *For $t > 1$, $E[q^B_t|\{\mathbb{H}_{t-1}, R_t = 1\}] \geq E[q^B_t|\{\mathbb{H}_{t-1}, R_t = 0\}] \geq E[q^B_t|\{\mathbb{H}_{t-1}, R_t = -1\}]$, where the inequalities are strict if $\mathbb{H}_{t-1}$ only includes positive ratings.*
2. ***Ratings are valuable:*** *$p_{\mathbb{H}_3} = E[q^B_3|\mathbb{H}_3] - q^A > 0$ if $\mathbb{H}_t$ only contains positive ratings, and for $t < 3$, we have $p_{\mathbb{H}_t} = E[q^B_t|\mathbb{H}_t, p_{\mathbb{H}_t}] - q^A > 0$ if $\mathbb{H}_t$ only contains positive ratings. Otherwise, $p_{\mathbb{H}_t} = 0$ and firm A sells in period t.*

Furthermore, in period $t < 3$ for any history $\mathbb{H}_t = \{\emptyset, \{1\}\}$:

3. ***Firm** A sets $p^A_t = 0$ and faces no demand.*
4. ***Firm** h charges $\overline{p}_{\mathbb{H}_t} = E[q^B_t|\mathbb{H}_t, \overline{p}_{\mathbb{H}_t}] - q^A$ with probability 1 and receives $R_t \in \{0, 1\}$.*
5. ***Firm** l either charges a mixed-strategy equilibrium such that:*
   (a) It charges $\overline{p}_{\mathbb{H}_t} > 0$ with probability $\delta^*_{\mathbb{H}_t}$ and receives $R_t \in \{-1, 0\}$, where $\delta^*_{\mathbb{H}_t} \in (\underline{\delta}_t, 1]$.
   (b) It charges $\underline{p}_{\mathbb{H}_t} = -q^A < 0$ with probability $1 - \delta^*_{\mathbb{H}_t}$ and receives $R_t \in \{0, 1\}$.
   Otherwise, if this mixed-strategy equilibrium does not exist, firm l sets $\delta^*_{\mathbb{H}_t} = 1$ such that $\overline{p}_{\mathbb{H}_t} < 0$ and receives either $R_t = 1$ or $R_t = 0$.

Towards proving this proposition, we will show a range of lemmas and corollaries that follow along the lines of the proof of Proposition 4. Recall that Lemmas 3 to 9 and Corollary 5 to 8 hold also for the three period model.

Lemma 18. *Given $\max\{E[q^B_3|\{1, 0\}], E[q^B_2|\{0\}]\} < q^A$ such that firm B sells if and only if $\mathbb{H} = \{\emptyset, \{1\}, \{1, 1\}\}$:*

The period 2 beliefs if both types of newcomers enter in period 1 and 2 are

$$E[q^B_2|\mathbb{H}_1, p_2] = \begin{cases} \frac{\gamma(1-F(q^h-\overline{p}_\emptyset))q^h}{\gamma(1-F(q^h-\overline{p}_\emptyset))+(1-\gamma)\delta^*_{\{0\}}(1-F(|\overline{p}_\emptyset|))} & \text{for } \mathbb{H}_1 = \{0\},\ \forall p_2 = \overline{p}_{\{0\}} \text{ if } \overline{p}_\emptyset \leq 0 \\ \frac{\gamma(1-F(q^h-\overline{p}_\emptyset))q^h}{\gamma(1-F(q^h-\overline{p}_\emptyset))+(1-\gamma)\delta^*_{\{0\}}\left[\delta^*_\emptyset(1-F(\overline{p}_\emptyset))+(1-\delta^*_\emptyset)(1-F(q^A))\right]} & \text{for } \mathbb{H}_1 = \{0\},\ \forall p_2 = \overline{p}_{\{0\}} \text{ if } \overline{p}_\emptyset > 0 \\ \frac{\gamma F(q^h-\overline{p}_\emptyset)q^h}{\gamma F(q^h-\overline{p}_\emptyset)+(1-\gamma)\delta^*_{\{1\}}F(|\overline{p}_\emptyset|)} & \text{for } \mathbb{H}_1 = \{1\},\ \forall p_2 = \overline{p}_{\{1\}} \text{ if } \overline{p}_\emptyset \leq 0 \\ \frac{\gamma F(q^h-\overline{p}_\emptyset)q^h}{\gamma F(q^h-\overline{p}_\emptyset)+(1-\gamma)(1-\delta^*_\emptyset)\delta^*_{\{1\}}F(q^A)} & \text{for } \mathbb{H}_1 = \{1\},\ \forall p_2 = \overline{p}_{\{1\}} \text{ if } \overline{p}_\emptyset > 0 \\ 0 & \text{otherwise.} \end{cases}$$

And the period 3 beliefs, if both types of newcomers entered in period 1 and 2, are

$$E[q_3^B|\mathbb{H}_2, p_3] =$$
$$\begin{cases} \frac{\gamma F(q^h-\overline{p}_\emptyset)(1-F(q^h-\overline{p}_{\{1\}}))q^h}{\gamma F(q^h-\overline{p}_\emptyset)(1-F(q^h-\overline{p}_{\{1\}}))+(1-\gamma)F(|\overline{p}_\emptyset|)(1-F(|\overline{p}_{\{1\}}|))} & \\ & \textit{for } \mathbb{H}_2=\{1,0\},\ \forall p_3 \ \textit{if } \overline{p}_\emptyset, \overline{p}_{\{1\}} \leq 0 \\ \frac{\gamma F(q^h-\overline{p}_\emptyset)(1-F(q^h-\overline{p}_{\{1\}}))q^h}{\gamma F(q^h-\overline{p}_\emptyset)(1-F(q^h-\overline{p}_{\{1\}}))+(1-\gamma)(1-\delta^*_\emptyset)F(q^A)(1-F(|\overline{p}_{\{1\}}|))} & \\ & \textit{for } \mathbb{H}_2=\{1,0\},\ \forall p_3 \ \textit{if } \overline{p}_\emptyset>0,\ \overline{p}_{\{1\}} \leq 0 \\ \frac{\gamma F(q^h-\overline{p}_\emptyset)(1-F(q^h-\overline{p}_{\{1\}}))q^h}{\gamma F(q^h-\overline{p}_\emptyset)(1-F(q^h-\overline{p}_{\{1\}}))+(1-\gamma)F(|\overline{p}_\emptyset|)\left[\delta^*_{\{1\}}(1-F(\overline{p}_{\{1\}}))+(1-\delta^*_{\{1\}})(1-F(q^A))\right]} & \\ & \textit{for } \mathbb{H}_2=\{1,0\},\ \forall p_3 \ \textit{if } \overline{p}_\emptyset\leq 0,\ \overline{p}_{\{1\}} > 0 \\ \frac{\gamma F(q^h-\overline{p}_\emptyset)(1-F(q^h-\overline{p}_{\{1\}}))q^h}{\gamma F(q^h-\overline{p}_\emptyset)(1-F(q^h-\overline{p}_{\{1\}}))+(1-\gamma)(1-\delta^*_\emptyset)F(q^A)\left[\delta^*_{\{1\}}(1-F(\overline{p}_{\{1\}}))+(1-\delta^*_{\{1\}})(1-F(q^A))\right]} & \\ & \textit{for } \mathbb{H}_2=\{1,0\},\ \forall p_3 \ \textit{if } \overline{p}_\emptyset, \overline{p}_{\{1\}} > 0 \\ \frac{\gamma F(q^h-\overline{p}_\emptyset)F(q^h-\overline{p}_{\{1\}})q^h}{\gamma F(q^h-\overline{p}_\emptyset)F(q^h-\overline{p}_{\{1\}})+(1-\gamma)F(|\overline{p}_\emptyset|)F(|\overline{p}_{\{1\}}|)} & \\ & \textit{for } \mathbb{H}_2=\{1,1\},\ \forall p_3 \ \textit{if } \overline{p}_\emptyset, \overline{p}_{\{1\}} \leq 0 \\ \frac{\gamma F(q^h-\overline{p}_\emptyset)F(q^h-\overline{p}_{\{1\}})q^h}{\gamma F(q^h-\overline{p}_\emptyset)F(q^h-\overline{p}_{\{1\}})+(1-\gamma)(1-\delta^*_\emptyset)F(q^A)F(|\overline{p}_{\{1\}}|)} & \\ & \textit{for } \mathbb{H}_2=\{1,1\},\ \forall p_3 \ \textit{if } \overline{p}_\emptyset>0,\ \overline{p}_{\{1\}} \leq 0 \\ \frac{\gamma F(q^h-\overline{p}_\emptyset)F(q^h-\overline{p}_{\{1\}})q^h}{\gamma F(q^h-\overline{p}_\emptyset)F(q^h-\overline{p}_{\{1\}})+(1-\gamma)F(|\overline{p}_\emptyset|)(1-\delta^*_{\{1\}})F(q^A)} & \\ & \textit{for } \mathbb{H}_2=\{1,1\},\ \forall p_3 \ \textit{if } \overline{p}_\emptyset\leq 0,\ \overline{p}_{\{1\}} > 0 \\ \frac{\gamma F(q^h-\overline{p}_\emptyset)F(q^h-\overline{p}_{\{1\}})q^h}{\gamma F(q^h-\overline{p}_\emptyset)F(q^h-\overline{p}_{\{1\}})+(1-\gamma)(1-\delta^*_\emptyset)F(q^A)(1-\delta^*_{\{1\}})F(q^A)} & \\ & \textit{for } \mathbb{H}_2=\{1,1\},\ \forall p_3 \ \textit{if } \overline{p}_\emptyset, \overline{p}_{\{1\}} > 0 \\ 0 & \textit{for } \mathbb{H}_2=\{1,-1\},\ \forall p_3 \\ E[q_2^B|\mathbb{H}_1, p_2] & \textit{otherwise.} \end{cases}$$

where $\delta^*_\emptyset$ is the mixed strategy in the first period, $\delta^*_{\mathbb{H}_t}$ is the mixed strategy in the period t following a history $\mathbb{H}_t$. And $\overline{p}_{\mathbb{H}_t}$ and $\underline{p}_{\mathbb{H}_t}$ are the prices following a history $\mathbb{H}_t$.

After histories where not both types of newcomers enter, beliefs are $q^l = 0$.

Proof of Lemma 18.
We now explain how we apply Bayes Rule to construct the above beliefs.

Recall from Corollary 8 that $\underline{p}_{\mathbb{H}_t} = -q^A$.

Corollary 7 implies that for any history which includes -1, consumer beliefs are q^l. Such histories are $\mathbb{H} = \{\{-1\}, \{1,-1\}, \{0,-1\}, \{-1,1\}, \{-1,0\}, \{-1,-1\}\}$.

We focus on the histories where firm B does not sell following no rating. In other words, following no rating, any subsequent beliefs must be the same as the previous period. Hence, following $\mathbb{H}_1 = 0$, it must be that beliefs are unchanged (for $\mathbb{H}_2 = \{\{0,1\},\{0,0\},\{0,-1\}\}$).

In any equilibrium where $\overline{p}_{\mathbb{H}_t} \leq 0$, Lemma 7 shows that $\delta^*_{\mathbb{H}_t} = 1$.

We now construct beliefs for $\mathbb{H}_1 = \{\{0\},\{1\}\}$. Suppose $\overline{p}_\emptyset \leq 0$, then $\delta^*_\emptyset = 1$ such that low-quality firms may only get no rating or a good rating. If the low-quality firm gets $R_1 = 1$, it does so with probability $F(|\overline{p}_\emptyset|)$. High-quality firms get a good rating with probability $F(q^h - \overline{p}_\emptyset)$. Low-quality firms only play $\overline{p}_{\{1\}}$ with some probability $\delta^*_{\{1\}}$, otherwise they play $\underline{p}_{\{1\}}$.

If, instead, the low-quality firm gets $R_1 = 0$, it does so with complementary probability $1 - F(|\overline{p}_\emptyset|)$. Likewise, the high-quality firm gets $R_1 = 0$ with complementary probability $1 - F(q^h - \overline{p}_\emptyset)$. Following $R_1 = 0$, low-quality firms only play $\overline{p}_{\{0\}}$ with some probability $\delta^*_{\{0\}}$, or they play $\underline{p}_{\{0\}}$ with probability $1 - \delta^*_{\{0\}}$.

Suppose instead that $\overline{p}_\emptyset > 0$, then $\delta^*_\emptyset \in (0,1)$. From Lemmas 6 and 7, high-quality firm B plays $\overline{p}_\emptyset$ with probability 1, obtaining a good rating with probability $F(q^h - \overline{p}_\emptyset)$. Low-quality firm B mixes between $\overline{p}_\emptyset$ and $\underline{p}_\emptyset$ with probabilities $\delta^*_\emptyset$ and $1 - \delta^*_\emptyset$, respectively. The low-quality firm may only obtain $R_1 = 1$ with probability $F(q^A)$ if it plays $\underline{p}_\emptyset$, which it does with probability $1 - \delta^*_\emptyset$.

Consider instead when firms get the rating $R_1 = 0$. The high-quality firm gets this rating with probability $(1 - F(q^h - \overline{p}_\emptyset))$, and l with probability $\left[\delta^*_\emptyset(1 - F(\overline{p}_\emptyset)) + (1 - \delta^*_\emptyset)(1 - F(q^A))\right]$.

This characterizes all period 2 beliefs.

We now characterize beliefs in period 3 following $\mathbb{H}_2 = \{1,1\}$. If prices are $\overline{p}_\emptyset > 0$ and $\overline{p}_{\{1\}} \leq 0$. Firm h gets these ratings with probability $F(q^h - \overline{p}_\emptyset)F(q^h - \overline{p}_{\{1\}})$, and l if it charges a low price in period 1 an draws sufficiently low rating effort, i.e. $(1 - \delta^*_\emptyset)F(q^A)F(|\overline{p}_{\{1\}}|)$.

If $\overline{p}_\emptyset, \overline{p}_{\{1\}} \leq 0$, then the high-quality firm obtains a sequence of good ratings with probability $F(q^h - \overline{p}_\emptyset) \cdot F(q^h - \overline{p}_{\{1\}})$, and the low-quality firm sets the price $\overline{p}_{\mathbb{H}_t}$ with probability 1 in each period and obtains a sequence of good ratings with probability $F(|\overline{p}_\emptyset|) \cdot F(|\overline{p}_{\{1\}}|)$.

If $\overline{p}_\emptyset \leq 0$ and $\overline{p}_{\{1\}} > 0$ such that $\delta^*_\emptyset = 1$. Then the high-quality firm obtains a sequence of good ratings with probability $F(q^h - \overline{p}_\emptyset) \cdot F(q^h - \overline{p}_{\{1\}})$, and the low-quality firm sets the price $\overline{p}_\emptyset$ in the first period, obtaining $R_1 = 1$ with probability $F(|\overline{p}_\emptyset|)$ and in the second period sets $\underline{p}_{\{1\}}$ with probability $(1 - \delta^*_{\{1\}})$ and obtains a good rating with probability $F(q^A)$.

If $\overline{p}_\emptyset > 0$ and $\overline{p}_{\{1\}} > 0$. Then the high-quality firm obtains a sequence of good ratings with probability $F(q^h - \overline{p}_\emptyset) \cdot F(q^h - \overline{p}_{\{1\}})$, and the low-quality firm sets the price $\underline{p}_\emptyset$ with probability $(1 - \delta^*_\emptyset)$ in the first period, obtaining $R_1 = 1$ with probability $F(q^A)$ and in the second period sets $\underline{p}_{\{1\}}$ with probability $(1 - \delta^*_{\{1\}})$ and obtains a good rating with probability $F(q^A)$.

Next, we characterize beliefs in the in period 3 following $\mathbb{H} = \{1,0\}$. If $\overline{p}_\emptyset > 0$ and $\overline{p}_{\{1\}} \leq 0$, firm h gets these ratings with probability $F(q^h - \overline{p}_\emptyset)(1 - F(q^h - \overline{p}_{\{1\}}))$. Firm l sets a low price in period 1, leading to $(1 - \delta^*_\emptyset)F(q^A)(1 - F(|\overline{p}_{\{1\}}|))$.

If $\overline{p}_\emptyset, \overline{p}_{\{1\}} \leq 0$, then the high-quality firm obtains a good rating in the first period with probability $F(q^h - \overline{p}_\emptyset)$ and no rating in the second period with probability $(1 - F(q^h - \overline{p}_{\{1\}}))$. The low-quality firm sets the price $\overline{p}_{\mathbb{H}_t}$ with probability 1 in both periods, obtaining a good rating in period 1 with probability $F(|\overline{p}_\emptyset|)$ and no rating in period 2 with probability $(1 - F(|\overline{p}_{\{1\}}|))$.

If $\overline{p}_\emptyset \leq 0$ and $\overline{p}_{\{1\}} > 0$ such that $\delta^*_\emptyset = 1$. Then the high-quality firm obtains a good rating in the first period with probability $F(q^h - \overline{p}_\emptyset)$ and no rating in the second period with probability $(1 - F(q^h - \overline{p}_{\{1\}}))$. The low-quality firm sets the price $\overline{p}_\emptyset$ in the first period with probability 1, obtaining $R_1 = 1$ with probability $F(|\overline{p}_\emptyset|)$. In the second period, the low-quality firm sets $\underline{p}_{\{1\}}$ with probability $(1 - \delta^*_{\{1\}})$ and obtains no rating with probability $(1 - F(q^A)$. Additionally, it sets the price $\overline{p}_{\{1\}}$ with probability $\delta^*_{\{1\}}$ and obtains no rating with probability $(1 - F(\overline{p}_{\{1\}}))$.

If $\overline{p}_\emptyset > 0$ and $\overline{p}_{\{1\}} > 0$, the high-quality firm obtains a good rating in the first period with probability $F(q^h - \overline{p}_\emptyset)$ and no rating in the second period with probability $(1 - F(q^h - \overline{p}_{\{1\}}))$. The low-quality firm sets the price $\underline{p}_\emptyset$ with probability $(1 - \delta^*_\emptyset)$ in the first period, obtaining $R_1 = 1$ with probability $F(q^A)$ and in the second period sets $\underline{p}_{\{1\}}$ with probability $(1 - \delta^*_{\{1\}})$ and obtains no rating with probability $(1 - F(q^A))$. Additionally, it sets the price $\overline{p}_{\{1\}} > 0$ with probability $\delta^*_{\{1\}}$ and obtains no rating with probability $(1 - F(\overline{p}_{\{1\}}))$.

If not both newcomers enter either in period 1 or 2, these histories are off-equilibrium and we set beliefs to q^l. By Corollary 5, histories where only the low-quality newcomer enters are off-path. Similarly, if only the high-quality firm enters, beliefs about newcomers are q^h and the newcomer sells at $q^h - q^A$ in each subsequent period. But then l has an incentive to deviate and enter and charge $q^h - q^A$ in the period where they enter, a contradiction. We conclude that histories after which not both types enter are off-equilibrium.

This characterizes consumer beliefs in every period following every history $\mathbb{H}$ given firm B sells only after a history where both types of newcomers enter and sell only after positive ratings.

We now find the condition where firm B does not sell in any history following at least a single instance of no rating or a bad rating. To do so, we find the most optimistic history following at least a single instance of no rating or a bad rating. Then we find the condition for which that belief is worse than q^A—such that consumers prefer to purchase from firm A instead of firm B following this history.

To find the most optimistic history, note that any such history cannot include $R_t = -1$. Following any history with $R_t = -1$, beliefs are $q^l = 0$ which are the most pessimistic beliefs possible. Next, note that the histories $\mathbb{H}_t = \{\{0, 0\}, \{0, 1\}\}$ are off-path as firm B does not sell following $\mathbb{H}_t = \{0\}$. We fix these beliefs equal $q^l = 0$. Moreover, since there are no sales on the path-of-play following $\{0\}$, this implies that $\overline{p}_{\{0\}} = \underline{p}_{\{0\}} = 0$, which implies $\delta^*_{\{0\}} = 1$. Given these statements, we know the most optimistic beliefs following a history which includes no rating or a bad rating must be either $\{0\}$ or $\{1, 0\}$. Thus, $\max\{E[q^B_3|\{1, 0\}], E[q^B_2|\{0\}]\} < q^A$ implies that if both newcomers enter, they sell only after histories with no ratings if $\max\{E[q^B_3|\{1, 0\}], E[q^B_2|\{0\}]\} < q^A$ holds. □

Lemma 19. *Suppose* $\max\{E[q_3^B|\{1,0\}], E[q_2^B|\{0\}]\} < q^A$ *holds and* f *is sufficiently flat such that* (1), (2) *and* (3) *hold. Then for all* $\mathbb{H}_t$ *on the path of play, there exists a unique* $\delta^*_{\mathbb{H}_t}$ *such that in period 3, firms charge* $p_3 = \max\{E[q_3^B|\mathbb{H}_t] - q^A, 0\}$. *Additionally, for all* $\mathbb{H}_t$ *on the path of play.*

1. *If* $\overline{p}_\emptyset > 0$ *and* (14) *hold, then* $\delta^*_\emptyset \in (0,1)$.
2. *If* $\overline{p}_\emptyset > 0$, (14), $\overline{p}_{\{1\}} > 0$ *and* (13) *hold, then* $\delta^*_{\{1\}} \in (0,1)$.
3. *If* $\overline{p}_\emptyset > 0$ *and* (14) *hold, and either* (13) *is violated or* $\overline{p}_{\{1\}} \leq 0$, *then* $\delta^*_{\{1\}} = 1$.
4. *If* $\overline{p}_\emptyset \leq 0$, *then* $\delta^*_\emptyset = 1$, *and the low-quality firm* B *obtains good ratings with some positive probability.*
5. *If* $\overline{p}_\emptyset \leq 0$, *then* $\overline{p}_{\{1\}} > 0$ *and* (13) *holds, then* $\delta^*_{\{1\}} \in (0,1)$.
6. *If* $\overline{p}_\emptyset \leq 0$, *and either* (13) *is violated or* $\overline{p}_{\{1\}} \leq 0$, *then* $\delta^*_{\{1\}} = 1$.

For all other histories, newcomers charge a price at marginal cost.

There exist parameters where entry does or does not occur. Indeed, if $q^A \to q^h$, *newcomers will not sell even after a positive rating, implying that newcomers never enter. However, if* q^h *is sufficiently large, reputation becomes increasingly valuable so that newcomers sell after a good rating and enter.*

Proof of Lemma 19.
We proceed as follows. We start by looking at the low-quality firm B's strategy in the third period, then in the second period, and finally in the first period.

In the third period, our Selection Assumption 2 implies that a low-quality firm B sets the highest possible positive price at which it would sell. This is because the third period is the terminal period and there is no continuation reputation effect that the firm needs to consider. Hence, to maximize profits it sets the highest price at which it sells with probability 1, given this price is positive.

In the second period, by Lemma 18, $\max\{E[q_3^B|\{1,0\}], E[q_2^B|\{0\}]\} < q^A$ implies that a low-quality firm B only sells if it has a history of $\mathbb{H}_t = \{1\}$. By Lemma 7, when the low-quality firm B sells, it mixes between $\overline{p}_{\{1\}}$ and $\underline{p}_{\{1\}}$. If the low-quality firm sets the price $\overline{p}_{\{1\}} = E[q_2^B|\{1\}, \overline{p}_{\{1\}}] - q^A > 0$, it makes the expected profits of $\overline{p}_{\{1\}} + 0$. The firm makes no continuation profit since we focus on the scenario where the firm B sells if it only has a history of good ratings. If the low-quality firm B sets $\underline{p}_{\{1\}} = -q^A$, it makes the expected profits of $\underline{p}_{\{1\}} + F(q^A)(E[q_2^B|\{1,1\}] - q^A)$ as it only sells if it obtains a good rating in period 3 with probability $F(q^A)$. Hence, $\delta^*_{\{1\}}$ makes the firm indifferent between these two choices. To find the unique $\delta^*_{\{1\}}$, we first show that the continuation profits after setting $\overline{p}_{\{1\}}$ is strictly decreasing in $\delta^*_{\{1\}}$ and the continuation profits after setting $\underline{p}_{\{1\}}$ is strictly increasing in $\delta^*_{\{1\}}$. Then we show that when $\delta^*_{\{1\}} = 0$, the continuation profits after setting $\overline{p}_{\{1\}}$ is strictly greater than setting $\underline{p}_{\{1\}}$, which implies that $\delta^*_{\{1\}} > 0$. Finally, we fix $\delta^*_{\{1\}} = 1$ and find the conditions for which an interior solution exists, failing which $\delta^*_{\{1\}} = 1$.

To see that the continuation profits after $\overline{p}_{\{1\}}$ is strictly decreasing in $\delta^*_{\{1\}}$, note the derivative of the continuation profits is

$$-\frac{\gamma F(q^h - \overline{p}_\emptyset)(1-\gamma)(1-\delta^*_\emptyset)F(q^A)q^h}{(\gamma F(q^h - \overline{p}_\emptyset) + (1-\gamma)(1-\delta^*_\emptyset)F(q^A)\delta^*_{\{1\}})^2} < 0.$$

To see that the continuation profits after $\underline{p}_{\{1\}}$ are strictly increasing in $\delta^*_{\{1\}}$, note first that $\underline{p}_{\{1\}} = -q^A$, and second, for the case $\overline{p}_\emptyset > 0$, the expectation term in period 3 is

$$\frac{\gamma F(q^h - \overline{p}_\emptyset)F(q^h - \overline{p}_{\{1\}})q^h}{\gamma F(q^h - \overline{p}_\emptyset)F(q^h - \overline{p}_{\{1\}}) + (1-\gamma)(1-\delta^*_\emptyset)F(q^A)(1-\delta^*_{\{1\}})F(q^A)}.$$

Since by the previous argument, $\overline{p}_{\{1\}}$ decreases in $\delta^*_{\{1\}}$, this expression strictly increases in $\delta^*_{\{1\}}$. Similarly, for the case $\overline{p}_\emptyset \leq 0$, the expectation term in period 3 is

$$\frac{\gamma F(q^h - \overline{p}_\emptyset)F(q^h - \overline{p}_{\{1\}})q^h}{\gamma F(q^h - \overline{p}_\emptyset)F(q^h - \overline{p}_{\{1\}}) + (1-\gamma)F(|\overline{p}_\emptyset|)(1-\delta^*_{\{1\}})F(q^A)},$$

which is strictly increasing for the same reason.

Then let $\delta^*_{\{1\}} = 0$, the continuation profits after $\overline{p}_{\{1\}}$ becomes $q^h - q^A$ and, for $\overline{p}_\emptyset > 0$, the continuation profits after $\underline{p}_{\{1\}}$ becomes $-q^A + F(q^A)\left[\frac{\gamma F(q^h - \overline{p}_\emptyset)F(q^h - \overline{p}_{\{1\}})q^h}{\gamma F(q^h - \overline{p}_\emptyset)F(q^h - \overline{p}_{\{1\}}) + (1-\gamma)(1-\delta^*_\emptyset)F(q^A)F(q^A)} - q^A\right]$. This is strictly smaller than $q^h - q^A$, since

$$\begin{aligned} q^h - q^A &> \frac{\gamma F(q^h - \overline{p}_\emptyset)F(q^h - \overline{p}_{\{1\}})q^h}{\gamma F(q^h - \overline{p}_\emptyset)F(q^h - \overline{p}_{\{1\}}) + (1-\gamma)(1-\delta^*_\emptyset)F(q^A)F(q^A)} - q^A \\ &> F(q^A)\left[\frac{\gamma F(q^h - \overline{p}_\emptyset)F(q^h - \overline{p}_{\{1\}})q^h}{\gamma F(q^h - \overline{p}_\emptyset)F(q^h - \overline{p}_{\{1\}}) + (1-\gamma)(1-\delta^*_\emptyset)F(q^A)F(q^A)} - q^A\right]. \end{aligned}$$

The argument for $\overline{p}_\emptyset \leq 0$ follows directly from the same steps.

Finally, let $\delta^*_{\{1\}} = 1$. Then the continuation profits after $\overline{p}_{\{1\}}$ becomes $\frac{\gamma F(q^h - \overline{p}_\emptyset)q^h}{\gamma F(q^h - \overline{p}_\emptyset) + (1-\gamma)(1-\delta^*_\emptyset)F(q^A)} - q^A$ and the continuation profits after $\underline{p}_{\{1\}}$, for $\overline{p}_\emptyset > 0$, become $-q^A + F(q^A)\left[q^h - q^A\right]$. Then if the continuation profits after $\overline{p}_{\{1\}}$ is still larger than the continuation profits after $\underline{p}_{\{1\}}$, we have $\delta^*_{\{1\}} = 1$. The interior solution exists if and only if

$$\begin{aligned} \frac{\gamma F(q^h - \overline{p}_\emptyset)q^h}{\gamma F(q^h - \overline{p}_\emptyset) + (1-\gamma)(1-\delta^*_\emptyset)F(q^A)} - q^A &< -q^A + F(q^A)\left[q^h - q^A\right] \\ \frac{\gamma F(q^h - \overline{p}_\emptyset)q^h}{\gamma F(q^h - \overline{p}_\emptyset) + (1-\gamma)(1-\delta^*_\emptyset)F(q^A)} &< +F(q^A)(q^h - q^A) \\ \frac{\gamma F(q^h - \overline{p}_\emptyset)q^h}{\gamma F(q^h - \overline{p}_\emptyset) + (1-\gamma)(1-\delta^*_\emptyset)F(q^A)} &< F(q^A)(q^h - q^A). \end{aligned} \tag{13}$$

Following the same argument for $\overline{p}_\emptyset \leq 0$ shows that the same condition applies to this case.

We now turn our attention to period 1.

In the first period the low-quality firm B sets $\overline{p}_\emptyset$ with strictly positive probability and obtains a continuation profit of $\frac{\gamma q^h}{\gamma+(1-\gamma)\delta^*_\emptyset} - q^A$. Otherwise, it may set $\underline{p}_\emptyset$ and make a continuation profit of

$$-q^A + F(q^A)\left(\frac{\gamma F(q^h - \overline{p}_\emptyset)q^h}{\gamma F(q^h - \overline{p}_\emptyset) + (1-\gamma)(1-\delta^*_\emptyset)F(q^A)\delta^*_{\{1\}}} - q^A\right).$$

Note that firm l only sets $\underline{p}_\emptyset$ with strictly positive probability if $\overline{p}_\emptyset > 0$.

Applying the same procedure, we show that the continuation profits after $\overline{p}_\emptyset$ is strictly decreasing in $\delta^*_\emptyset$ and the continuation profits after $\underline{p}_\emptyset$ is strictly increasing, implying that any $\delta^*_\emptyset$ is unique. Then allowing $\delta^*_\emptyset = 0$ we show that the continuation profits after setting $\overline{p}_\emptyset$ is strictly greater than setting $\underline{p}_\emptyset$, which tells us $\delta^*_\emptyset > 0$. We then let $\delta^*_\emptyset = 1$ and find the conditions for which an interior solution exists, failing which $\delta^*_\emptyset = 1$.

To see the continuation profits after $\overline{p}_\emptyset$ is strictly decreasing in $\delta^*_\emptyset$, note the derivative of the continuation profits is

$$-\frac{\gamma(1-\gamma)q^h}{(\gamma+(1-\gamma)\delta^*_\emptyset)^2} < 0.$$

To see the continuation profits after $\underline{p}_\emptyset$ is strictly increasing in $\delta^*_\emptyset$, note that by the previous argument, $\overline{p}_\emptyset$ decreases in $\delta^*_\emptyset$, implying that these continuation profits strictly increase in $\delta^*_\emptyset$.

Then for $\delta^*_\emptyset = 0$, the continuation profit from $\overline{p}_\emptyset$ becomes $q^h - q^A$, and the continuation profit from $\underline{p}_\emptyset$ becomes $-q^A + F(q^A)(\frac{\gamma F(q^h-\overline{p}_\emptyset)q^h}{\gamma F(q^h-\overline{p}_\emptyset)+(1-\gamma)F(q^A)\delta^*_{\{1\}}} - q^A)$. The continuation profits after a high price are strictly larger, since

$$\begin{aligned} q^h - q^A &> \frac{\gamma F(q^h - \overline{p}_\emptyset)q^h}{\gamma F(q^h - \overline{p}_\emptyset) + (1-\gamma)F(q^A)\delta^*_{\{1\}}} - q^A \\ &> F(q^A)\left(\frac{\gamma F(q^h - \overline{p}_\emptyset)q^h}{\gamma F(q^h - \overline{p}_\emptyset) + (1-\gamma)F(q^A)\delta^*_{\{1\}}} - q^A\right). \end{aligned}$$

Finally, for $\delta^*_\emptyset = 1$, the continuation profits after $\overline{p}_\emptyset$ is γq^h, and the continuation profits from $\underline{p}_\emptyset$ becomes $-q^A + F(q^A)(q^h - q^A)$. Then if the continuation profits after $\overline{p}_\emptyset$ is still larger than the continuation profits after $\underline{p}_\emptyset$, we have $\delta^*_\emptyset = 1$. The interior solution exists if and only if

$$\begin{aligned} \gamma q^h - q^A &< -q^A + F(q^A)(q^h - q^A) \\ \gamma q^h &< +F(q^A)(q^h - q^A). \end{aligned} \tag{14}$$

We now address each of the statements in Lemma 19 in turn.

Statement 1. If $\overline{p}_\emptyset > 0$ and (14) hold, then there exists a unique solution which is interior in period 1, $\delta^*_\emptyset \in (0,1)$.

Statement 2. If, in addition to Statement 1, $\overline{p}_{\{1\}} > 0$ and (13) hold, then there exists a unique solution which is interior in period 2, $\delta^*_{\{1\}} \in (0,1)$.

Statement 3. If, in addition to Statement 1 we have $\overline{p}_{\{1\}} \leq 0$, then Lemma 7 implies $\delta^*_{\{1\}} = 1$. And if, in addition to Statement 1, (13) is violated, then $\delta^*_{\{1\}} = 1$.

Statement 4. If $\overline{p}_\emptyset \leq 0$, Lemma 7 implies $\delta^*_\emptyset = 1$. Additionally, because $\overline{p}_\emptyset \leq 0$, the low-quality firm B obtains good ratings with some positive probability.

Statement 5. If, in addition to Statement 4, $\overline{p}_{\{1\}} > 0$ and (13) holds, then there exists a unique solution which is interior in period 2, $\delta^*_{\{1\}} \in (0,1)$.

Statement 6. If, in addition to Statement 4, $\overline{p}_{\{1\}} \leq 0$ then by Lemma 7, we have $\delta^*_{\{1\}} = 1$. And if, in addition to Statement 4, (13) is violated, then $\delta^*_{\{1\}} = 1$.

These statements describe $\delta^*_{\mathbb{H}_t}$ after histories that occur in equilibrium with strictly positive probability and shows they are unique up to off-path-beliefs.

All other histories occur with probability zero on the path of play, so we set newcomer prices to marginal cost.

This concludes the proof. □

Lemma 20. *After any history $\mathbb{H}_t$, a high-quality firm B sells if and only if a low-quality firm B sells. If firm B sells in any period $t+1$, it also sells in period t.*

Suppose (17) *and* (18) *hold. Then there exist unique values $\underline{\delta}_\emptyset \in (0,1)$ and $\underline{\delta}_{\{1\}} \in (0,1)$ such that the following holds. Both types of newcomer B enter in period 1 if and only if $\delta^*_\emptyset \geq \underline{\delta}_\emptyset$; otherwise none enters. Both types of newcomer B enter in period 2 if and only if $\delta^*_\emptyset \geq \underline{\delta}_\emptyset$ and $\delta^*_{\{1\}} \geq \underline{\delta}_{\{1\}}$ hold; otherwise none enters in period 2.*

Proof of Lemma 20.
First, we know from Corollary 5 that if low-quality firm B sells, a high-quality firm B must also sell.

We now show that if h enters, then also l enters. Towards a contradiction, suppose only firm h enters. Then $E[q^B_t|p_t] = q^h \forall p_t$. For any rating, newcomers are identified as h, which allows the firm to always get the profit of $q^h - q^A$ in each period. But then l has a profitable deviation to enter the market with positive prices $q^h - q^A$ in period t, contradicting that l does not enter. We conclude that after any history, h enters if and only if l enters.

Suppose instead the high-quality firm B chooses to enter the market in some period t but not the period $t-1$. Then it faces the same payoffs as entering in $t-1$ as it is unable to build its reputation.

Thus if the high-quality firm chooses to enter the market in a period t rather than $t-1$, it forgoes the additional continuation profit it could make from a good reputation. Therefore, it must be that if the high-quality firm chooses to sell at all, it must sell in period 1.

Since, if the high-quality firm B sells at all it sells in period 1 and the low-quality firm B chooses to sell if the high-quality firm is selling, then it must be that if firm B enters the market it enters in the first period and this decision is independent of it's quality realization.

We now check for the conditions which ensure firm B sells after a history of good ratings. In other words, firm B has to be able to sell following a good rating, $E[q_t^B|\mathbb{H}_t, p_t] \geq q^A$ for $\mathbb{H}_t \in \{\{1\}, \{1,1\}\}$. Suppose to the contrary that $E[q_t^B|\mathbb{H}_t, p_t] < q^A$ for some $\mathbb{H}_t \in \{\{1\}, \{1,1\}\}$. Then it must be that firm B does not sell in period t. This means there is no incentive for the low-quality firm B to harvest ratings and set the low price in period $t-1$. Hence $\delta^*_{\mathbb{H}_t} = 1$ and firm B becomes inactive in period t. Therefore, firm B sells after a good rating if $E[q_t^B|\mathbb{H}_t, p_t] \geq q^A$ for $\mathbb{H}_t \in \{\{1\}, \{1,1\}\}$.

We now derive more precise conditions for selling, conditional on entry, for these separate histories.

We start with $\mathbb{H}_t = \{\{1\}\}$ Rearranging $E[q_t^B|\{1\}, p_t] > q^A$, implies that

$$\begin{aligned} E[q_2^B|\{1\}, p_2] > q^A \Leftrightarrow \\ \gamma F(q^h - \overline{p}_\emptyset)(q^h - q^A) > (1-\gamma)(1-\delta^*_\emptyset)\delta^*_{\{1\}} F(q^A) q^A \end{aligned} \tag{15}$$

where the left hand side is strictly increasing in $\delta^*_\emptyset$ and, since by Lemma 19 $\delta^*_{\{1\}} > 0$, the right hand side strictly decreasing in $\delta^*_\emptyset$. Therefore, if $\delta^*_\emptyset$ is sufficiently large, firm B sells after $\mathbb{H}_t = \{\{1\}\}$.

Next, consider $\mathbb{H}_t = \{1, 1\}$. Firm B sells if

$$\begin{aligned} E[q_3^B|\{1,1\}, p_3] > q^A \Leftrightarrow \\ \gamma F(q^h - \overline{p}_\emptyset) F(q^h - \overline{p}_{\{1\}})(q^h - q^A) > (1-\gamma)(1-\delta^*_\emptyset) F(q^A)^2 (1-\delta^*_{\{1\}}) q^A \end{aligned} \tag{16}$$

where the left hand side is strictly increasing in $\delta^*_{\{1\}}$ and the right hand side strictly decreasing if $\delta^*_\emptyset < 1$. If $\delta^*_\emptyset = 1$, the right had side is zero, and the condition always holds. Therefore, conditional on entry, firm B sells in period 3 if $\delta^*_{\{1\}}$ is sufficiently large.

We now characterize entry decisions in period 1 and 2. We start with period 1. Note that if $\overline{p}_\emptyset > 0$, then newcomers charge strictly positive prices in period 1 with probability $\delta^*_\emptyset > 0$, implying they earn strictly positive profits. If $\overline{p}_\emptyset \leq 0$, we know from Lemma 7 that $\delta^*_\emptyset = 1$ such that $\overline{p}_\emptyset < 0$. Then the following condition implies that firm l sells with strictly positive probability and earns strictly positive profits:

$$\gamma q^h - q^A + F(q^A - \gamma q^h)\left[\frac{\gamma F(q^A + (1-\gamma)q^h)q^h}{\gamma F(q^A+(1-\gamma)q^h) + (1-\gamma)F(q^A - \gamma q^h)} - q^A + F(|\overline{p}_{\{1\}}|)\left[\frac{\gamma F(q^h - \overline{p}_\emptyset)F(q^h - \overline{p}_{\{1\}})q^h}{\gamma F(q^h - \overline{p}_\emptyset)F(q^h - \overline{p}_{\{1\}}) + (1-\gamma)F(|\overline{p}_\emptyset|)F(|\overline{p}_{\{1\}}|)} - q^A\right]\right] > 0, \tag{17}$$

This is the profits when both $\overline{p}_\emptyset < 0$ and $\overline{p}_{\{1\}} < 0$. Clearly, since h must earn weakly larger profits, this implies that also h sells with strictly positive probability. We conclude that if (15) holds, both types of firm B have strictly positive demand in period 1 and therefore enter. Since they clearly do not enter if this condition is violated, and since by our above arguments, it holds with equality for a unique $\underline{\delta}_\emptyset \in (0,1)$, we conclude that there exists a unique $\underline{\delta}_\emptyset \in (0,1)$ such that all types of newcomers enter in period 1 if and only if $\delta^*_\emptyset \geq \underline{\delta}_\emptyset$.

We continue with period 2. Note that if $\overline{p}_{\{1\}} > 0$, then newcomers charge strictly positive prices in period 1 with probability $\delta^*_{\{1\}} > 0$, implying they earn strictly positive profits. If $\overline{p}_{\{1\}} \leq 0$, we know from Lemma 7 that $\delta^*_{\{1\}} = 1$ such that $\overline{p}_{\{1\}} < 0$. Then the following condition implies that firm l sells with strictly positive probability and earns strictly positive profits:

$$\gamma q^h - q^A + F(|\overline{p}_{\{1\}}|)\left[\frac{\gamma F(q^h - \overline{p}_\emptyset)F(q^h - \overline{p}_{\{1\}})q^h}{\gamma F(q^h - \overline{p}_\emptyset)F(q^h - \overline{p}_{\{1\}}) + (1-\gamma)F(|\overline{p}_\emptyset|)F(|\overline{p}_{\{1\}}|)} - q^A\right] > 0, \tag{18}$$

Firm B can only sell if it obtained a positive rating in period 1. Then firm l sells despite a negative price in period 2 if this condition holds. Thus, since firm h earns weakly larger profits, also firm h sells.

We conclude that if (17) and (18) hold, both types of newcomers enter in period 2 if and only if (15) and (16) hold. Since they clearly do not enter in period 2 if (16) is violated, and since by our above arguments, it holds with equality for a unique $\underline{\delta}_{\{1\}} \in (0,1)$, we conclude that there exists a unique $\underline{\delta}_{\{1\}} \in (0,1)$ such that all types of newcomers enter in period 2 if and only if $\delta^*_\emptyset \geq \underline{\delta}_\emptyset$ and $\delta^*_{\{1\}} \geq \underline{\delta}_{\{1\}}$.

Finally, by our Selection Assumption 1, the equilibria with entry are played whenever they exist. This concludes the proof. □

Proof of Proposition 6.

First, note that the claims on entry follow directly from Lemma 20.

Statement 1: Suppose $\max\{E[q^B_3|\{1,0\}], E[q^B_2|\{0\}]\} < q^A$. Then Lemma 18 shows that $E[q^B_t|\{\mathbb{H}_{t-1}, R_t = -1\}] = q^l = 0$ and $E[q^B_t|\{\mathbb{H}_{t-1}, R_t = 0\}] \in (0, q^A)$. Next, from Lemma 20 we know that $E[q^B_t|\mathbb{H}_t, p_t] > q^A$ for $\mathbb{H}_t = \{\{1\},\{1,1\}\}$ whenever firm B enters. Therefore, we know that

$E[q_t^B|\{\mathbb{H}_{t-1}, R_t = 1\}] > q^A > E[q_t^B|\{\mathbb{H}_{t-1}, R_t = 0\}]$. This shows that $E[q_t^B|\{\mathbb{H}_{t-1}, R_t = 1\}] > E[q_t^B|\{\mathbb{H}_{t-1}, R_t = 0\}] > E[q_t^B|\{\mathbb{H}_{t-1}, R_t = -1\}]$ if the history $\mathbb{H}_{t-1}$ includes only positive ratings. Now recall that if the history includes negative or no rating, then there is no sales and no updating. Hence, obtaining a rating in the period t does not change consumer expectations and $E[q_t^B|\{\mathbb{H}_{t-1}, R_t = 1\}] = E[q_t^B|\{\mathbb{H}_{t-1}, R_t = 0\}] = E[q_t^B|\{\mathbb{H}_{t-1}, R_t = -1\}]$.

Statement 2: $\max\{E[q_3^B|\{1, 0\}], E[q_2^B|\{0\}]\} < q^A$ implies that firm B does not sell if it receives either no or negative ratings. By Lemma 20, newcomers enter after a history of positive ratings. Then Lemma 19 characterized the prices in the statement.

Statement 3: Follows directly from Lemmas 19 and 20. Lemma 20 characterized which histories induce positive demand, and Lemma 19 characterizes the prices.

Statement 4: Comes directly from Corollary 6.

Statement 5: The strategies are characterized in Lemma 19. The price levels set by the firm B is follows from Corollary 8.

□

B.3 Comparative statics

As before, we focus on mixed-strategy equilibria for our comparative statics. Our arguments in Lemma 19 imply that such equilibria indeed exist for some parameters, i.e. if $\frac{F(q^A)(q^h - q^A)}{\gamma q^h}$ is sufficiently large.

Corollary 15. *Consider an increase in $\bar{e}$. In each period, the low-quality firm harvest ratings more, $\frac{\partial \delta^*_{\emptyset}}{\partial \bar{e}} > 0$ and $\frac{\partial \delta^*_{\{1\}}}{\partial \bar{e}} > 0$.*

Proof of Corollary 15.
In period 2, in equilibrium, the low-quality firm B is indifferent between the profit it obtains from setting $\overline{p}_{\{1\}}$ and $\underline{p}_{\{1\}}$. This means

$$\frac{\gamma(q^h - \overline{p}_{\emptyset})q^h}{\gamma(q^h - \overline{p}_{\emptyset}) + (1-\gamma)(1-\delta^*_{\emptyset})\delta^*_{\{1\}}q^A} = \frac{q^A}{\bar{e}}\left(\frac{\gamma(q^h - \overline{p}_{\emptyset})(q^h - \overline{p}_{\{1\}})q^h}{\gamma(q^h - \overline{p}_{\emptyset})(q^h - \overline{p}_{\{1\}}) + (1-\gamma)(1-\delta^*_{\emptyset})(1-\delta^*_{\{1\}})(q^A)^2} - q^A\right).$$

Then for a given $\delta^*_{\emptyset}$, it is immediate that setting a higher $\delta^*_{\{1\}}$ decreases the left side of this equation and changes in $\bar{e}$ only affects this equation through $\delta^*_{\{1\}}$.

On the right side, from Lemma 19 we know that the payoff $\frac{\gamma(q^h - \overline{p}_{\emptyset})(q^h - \overline{p}_{\{1\}})q^h}{\gamma(q^h - \overline{p}_{\emptyset})(q^h - \overline{p}_{\{1\}}) + (1-\gamma)(1-\delta^*_{\emptyset})(1-\delta^*_{\{1\}})(q^A)^2} - q^A$ is increasing in $\delta^*_{\{1\}}$. Further, fixing $\delta^*_{\{1\}}$, increases in $\bar{e}$ leads to a decrease in the probability of obtaining the period 3 payoff $\frac{\gamma(q^h - \overline{p}_{\emptyset})(q^h - \overline{p}_{\{1\}})q^h}{\gamma(q^h - \overline{p}_{\emptyset})(q^h - \overline{p}_{\{1\}}) + (1-\gamma)(1-\delta^*_{\emptyset})(1-\delta^*_{\{1\}})(q^A)^2} - q^A$.

Taken together, this means that following an increase in $\bar{e}$, the direct effect is the right side of the equation decreases because the probability of obtaining a period 3 payoff decreases. Hence, to compensate for this decrease and restore indifference, an increase in $\delta^*_{\{1\}}$ is required as this simultaneously decreases the left side of the equation and increases the period 3 payoff in the right side of the equation. Therefore, it follows that $\frac{\partial \delta^*_{\{1\}}}{\partial \overline{e}} > 0$.

We now turn our attention to the first period. In equilibrium, the low-quality firm is indifferent between the profit it obtains from setting $\overline{p}_\emptyset$ and $\underline{p}_\emptyset$. This means

$$\frac{\gamma q^h}{\gamma + (1-\gamma)\delta^*_\emptyset} = \frac{q^A}{\overline{e}}\left(\frac{\gamma(q^h - \overline{p}_\emptyset)q^h}{\gamma(q^h - \overline{p}_\emptyset) + (1-\gamma)(1-\delta^*_\emptyset)(q^A)\delta^*_{\{1\}}} - q^A\right).$$

Observe here that the left side of the equation is decreasing in $\delta^*_\emptyset$ and that changes in $\overline{e}$ affects the left side of the equation only through $\delta^*_\emptyset$. On the right side of the equation, increases in $\overline{e}$ has a direct effect of reducing the probability of obtaining a continuation payoff. Note that the side of the continuation payoff is increasing in $\delta^*_\emptyset$.

Hence, considering an increase in $\overline{e}$, the direct effect leads to a decrease to the right side of the equation. To restore indifference, an increase in $\delta^*_\emptyset$ decreases the left side of the equation and simultaneously increases the probability of the continuation payoff following a good rating (the terms in brackets on the right side of the equation). Therefore, to restore equilibrium, it must be that $\frac{\partial \delta^*_\emptyset}{\partial \overline{e}} > 0$.

□